Approximation of functions:
Problem Sheet 4 - Week 5 class

1. The following exercises from LNT's book: 9.2*, 10.3, 10.4, 10.5, 10.6, 10.10, 11.3

   Exercises with a * require chebfun computing.

2. 2007 Finals

   Let B be an inner product space of continuous functions defined on $[0, 1]$ with inner product

   $$(u, v) = \int_0^1 xu(x)v(x)\,\mathrm{d}x \qquad \forall u, v \in B$$

   and norm $||u||_2 = \sqrt{(u, u)}$. Let A be a finite dimensional subspace of B.

   (a) Prove that for any $u \in B$, $p \in A$ is the best L_2 approximation from A to u if, and only if,

   $$(u - p, q) = 0 \qquad \forall q \in A.$$

   [You may assume without proof that a best approximation exists.]

   (b) Use a Gram–Schmidt process to construct the three lowest-order orthogonal polynomials with respect to this norm.

   (c) Let $A = P_1$, the space of linear polynomials. Determine $p \in P_1$ that is the best L_2 approximation to the function u defined by

   $$u(x) = \sqrt{1 - x^2}, \qquad x \in [0, 1].$$