MSc Special Topic: Approximation of Functions - MT 2012

You are welcome to suggest any topic in approximation theory. The following are just some suggestions to illustrate the range of topics possible.

1. L_2 approximation

Consider approximation of $y(x) = \sqrt{1-x^2}$ on $-1 < x < 1$.

(a) Derive approximations using (i) Legendre and (ii) Chebyshev polynomials. Use matlab to investigate the error as the number of approximating polynomials increases.

(b) Read about and describe some of the background theory for approximation using rational polynomials. Consider approximation of y using rational polynomials where the denominator and numerator have degree two or three. Is this approximation better than using orthogonal polynomials?

2. Exchange algorithm

Look at the best minimax approximation of $y = \sqrt{x}$, $0 \leq x \leq 1$ using linear, quadratic and cubic polynomials. Can you derive analytic solutions for the best approximation? Implement a simple version of the exchange algorithm using matlab and investigate the rate of convergence to your analytic solutions for a few different functions.

3. Chebyshev polynomials

Investigate the use of Chebyshev polynomial expansions in determining approximate solutions of integral equations where there is a singularity in the kernel of the integrand. The aim here is to both describe some of the theory and to apply the theory to compute solutions for a problem of your choice. (see chapter 9 of Mason & Handscomb to start with)

4. More Chebyshev polynomials

Investigate the role played by Chebyshev polynomials in approximation of the inverse of matrices. (Perhaps start with Fischer, B. 'Polynomial based iteration methods for symmetric linear systems')

5. Rational approximation

Look into the theory of rational approximation and how you might develop an exchange type algorithm. Investigate some properties (such as rate of convergence) for two or three functions of your choice.

6. The chebfun system

Investigate deeper into the 'chebfun' and 'chebop' system. You could write about the theoretical background for the system or apply the system to a particular problem.

7. Computer Graphics

Look into how a surface is represented using Bezier patches. It turns out this is the same as using B-splines. Look up this theory and apply the de Casteljau algorithm to generating a surface. (start with Foley, van Dam, Feimer, Hughes & Phillips 'Introduction to Computer Graphics'). Implement a simple surface generator using Bezier patches in matlab and use matlab's lighting routines to give some perspective to the surface.

8. Lebesgue constants

Investigate Lebesgue constants, their history, importance and how they are computed. This could be a theoretical discussion or you could write matlab routines to calculate Lebesgue constants for operations such as Lagrange, Hermite or spline interpolation.

9. Jackson's Theorems

There are a number of important theorems associated with Jackson. Write about the background, the theorems and their place in approximation theory. Use the chebfun system to illustrate the theorems in action.

10. Approximations Theory and Approximations Practice

I am happy to discuss combining some of the advanced exercises in Nick Trefethen's book which we will not cover but which might fomr the basis of a special topic.